

A NEW INTEGRAL METHOD FOR THE KINETIC ANALYSIS OF THERMOGRAVIMETRIC DATA

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Methods for the calculation of activation energies, pre-exponential factors and reaction orders from thermogravimetric data are briefly reviewed. A new integral method is proposed for the determination of these kinetic parameters, using data from pairs of TG curves produced at different heating rates. Employing accurate values of the temperature integral of the Arrhenius equation, tabulated over a range of E and T , and a simple graphical procedure, the method offers advantages of speed and accuracy over those previously reported. It is suggested that at least one of the kinetic parameters should be allowed to move freely in order to achieve the best possible fit between calculated and experimental traces.

If the thermal decomposition of a material is accompanied by weight loss, the changes of weight can be followed by thermogravimetric measurements and, from the data produced, valuable information can be obtained concerning the thermal decomposition process itself.

During the course of the thermogravimetric studies the sample is heated at elevated temperatures and the weight loss is recorded as a function of time. A constant heating rate is usually employed, and in this case the time units can easily be transformed into temperature units.

To obtain kinetic data (activation energy, pre-exponential factor, reaction order) from the recorded curves, the well known kinetic equation:

$$dx/dt = Ae^{-E/RT} f(x) \quad (1)$$

or its modification containing the constant heating rate:

$$dx/dT = (A/a)e^{-E/RT} f(x) \quad (2)$$

serves as a basis for calculations, where $a = dT/dt$ is the linear heating rate, A the pre-exponential factor, E the activation energy, and x the fractional weight of the reacted material related to the total weight loss, while R , T and t are the gas constant, the absolute temperature and the time, respectively. The term $f(x)$ is a function of the weight loss and its analytical form depends on the mechanism of the reaction in question. In some cases $f(x)$ can be expressed as $(1-x)^n$ where n is the reaction order. It must be pointed out that in stepwise decomposition, such as occurs when, for example, calcium oxalate is heated, the above equations can be used only for analysis of the separate steps.

The evaluation of thermoanalytical data has been discussed in several review articles [1–3]. These show two distinct approaches for the determination of kinetic parameters from thermogravimetric measurements. Thus, the Arrhenius-type expressions (Eqs 1 and 2) may be treated either

(a) by integral analysis, which considers the expressions as differential equations and obtains the kinetic constants from their integrated forms, or

(b) by obtaining the data in a differential form and setting the values of dx/dt , x , and T into a suitably derived form of the expressions.

The choice between the two approaches is not simply a matter of taste since, in the differential approach, (b), the determination of the apparent reaction rates requires either a special derivative thermogravimetric (DTG) apparatus or a not too accurate and lengthy procedure of graphical differentiation of the TG curve, whereas when integral methods are used the weight loss-temperature curves are sufficient.

The main difficulties arising in the application of integral methods are that in the rearranged form of Eq. (2):

$$dx/f(x) = (A/a)e^{-E/RT} dT \quad (3)$$

neither the analytical form of $f(x)$ is known nor the right hand side of the expression can be integrated in a finite form.

The differences between the integral methods reported in the literature are those of the approach to, and the solution of, the integral equation:

$$F(x) = \int_{x_0}^x dx/f(x) = (A/a) \int_{T_0}^T e^{-E/RT} dT \quad (4)$$

where x_0 and T_0 are the corresponding values of the initial conversion and absolute temperature.

The methods suggested for the solution of Eq. (4) are of two different types as far as the integration of the right hand side is concerned. One group of authors [4–7] suggests approximate expressions for the exponential integral, while others [8–10] recommend the use of numerically integrated and tabulated values of it. Although the approximative methods are generally easier to use, for more accurate results those methods using the accurate integral values are to be preferred.

Doyle [8] has suggested a curve fitting method for analysing thermogravimetric data from decomposition reactions having a known analytical form of $f(x)$. He assumes that the values of $F(x)$, for the materials under consideration, are very small at low temperatures and substitutes 0 for T_0 . Thus:

$$(A/a) \int_{T_0}^T e^{-E/RT} dT = (A/a) \int_0^T e^{-E/RT} dT \quad (5)$$

By modifying Eq. (4) as shown in Eq. (5), and using the notation: $u = E/RT$, we obtain:

$$F(x) = (-AE/aR) \int_{\infty}^z (e^{-u}/u^2) du \quad (6)$$

where z is the value of u at the temperature corresponding to the apparent value of x .

Introducing the notation:

$$p(z) = - \int_{\infty}^z (e^{-u}/u^2) du \quad (7)$$

the equation of the TG curves is obtained:

$$F(x) = (AE/aR)p(z) \quad (8)$$

The values of $-\log p(z)$ have been given by Doyle for E/RT values ranging from 10 to 50.

Starting with an approximate value of E and an assumed analytical form for $F(x)$, an approximate $x-T$ curve can be calculated with the aid of the tabulated $-\log p(z)$ values and compared with the experimental curve. The procedure can be repeated with other E values until the possible best agreement of the calculated and experimental curves has been achieved and the best-fit E value thereby obtained.

A may be determined simultaneously with E , and is calculated on the basis of the equation of the TG curve slope. The main drawbacks of this method are that $f(x)$ must be known or assumed and that the procedure is lengthy.

A more convenient and elegant procedure has been suggested by Zsakó [9]. He rearranges Eq. (8) and takes logarithms to obtain the expression:

$$\log(AE/aR) = \log F(x) - \log p(z) = B \quad (9)$$

where B is a constant containing A , E , R and the heating rate, the last named remaining constant in a particular determination.

By assuming different reaction mechanisms and activation energies, series of values for B can be calculated using the tabulated values of $\log p(z)$, which have been given by Zsakó for different temperatures and activation energies. The validity of the assumptions can be characterised by the standard deviation of B . Thus the minimum standard deviation (maximum consistency) of B would show the best E value and the maximum consistency of the decomposition process with the assumed $f(x)$ function. With a knowledge of the $f(x)$ and E values the pre-exponential factor, A , can easily be determined. The main achievement of Zsakó's method is that it allows three characteristics of the decomposition process to be determined simultaneously. The required calculations are, however, still lengthy.

Satava and Skvara [10] have simplified Zsakó's treatment and use a graphical curve fitting method for the simultaneous determination of the $f(x)$ function and the values of E and A . The method is based upon Eq. (9). Values of $\log p(z)$ have been calculated and plotted against the absolute temperature for various activation energies. Log $F(x)$ values corresponding to various mechanisms have been calculated for a wide range of values of x and given in a tabulated form. Diagrams of $F(x)$ versus T are then constructed for assumed reaction mechanisms

and drawn on transparent paper. By shifting this paper along the $\log p(z)$ versus T diagram until the $\log p(z)$ and $\log F(x)$ graphs correspond perfectly, one is able to determine the activation energy and the $F(x)$ ($f(x)$) function.

More recently Satava [11] has described an improved method for determining the $f(x)$ function.

The advantage of the methods described by Satava and Skvara is that they allow for identification of curve portions characterised by different reaction mechanisms and/or activation energies. However, since the procedure used is based on curves corresponding to discrete E values and assumed $f(x)$ functions, it is hardly faster than that suggested by Zsakó.

In the present paper a new method is described for the kinetic analysis of thermogravimetric data. The method is more rapid and, in some aspects, more advantageous than those described above. In the method we use the notation:

$$i = \int_0^T e^{-E/RT} dT \quad (10)$$

Eq. (4) can be written:

$$F(x) = (A/a)i \quad (11)$$

For two points of the same conversion, x , but obtained from two different TG curves of the same material produced at different heating rates and from samples preferably of the same initial weight, the following expressions can be written:

$$i_{11} = \int_0^{T_{11}} e^{-E/RT} dT; \quad i_{21} = \int_0^{T_{21}} e^{-E/RT} dT \quad (12)$$

$$F(x)_{11} = (A/a_1)i_{11}; \quad F(x)_{21} = (A/a_2)i_{21} \quad (13)$$

$$F(x)_{11} = F(x)_{21} \quad (14)$$

where the first figure in the subscript indicates the TG curve and the second a particular conversion value. Combining (13) and (14) we can write:

$$F(x)_{11}/F(x)_{21} = (a_2/a_1)/(i_{11}/i_{21}) = 1 \quad (15)$$

and hence:

$$a_1/a_2 = i_{11}/i_{21} \quad (16)$$

or generally expressed:

$$a_k/a_1 = i_{k1}/i_{11} = i_{k2}/i_{12} = \dots = i_{kn}/i_{1n} \quad (17)$$

The i values can be calculated numerically and given in tabulated or diagrammatic forms. For calculation purposes it is more convenient to have values of $\log i$.

For determination of the activation energy, the method is applied as follows: two points of the same conversion are chosen on two TG curves obtained at different heating rates (a_1 and a_2) and the corresponding temperatures, T_{11} and T_{21} noted. The values of $\log i$ corresponding to these temperatures at various

activation energies can be obtained from the tabulated values. If we plot $\log(i_1/i_2)$ (here the subscripts 1 and 2 refer to the values corresponding with T_{11} and T_{21}) against E , the unknown E can be located as that value of E corresponding to $\log(a_1/a_2) = \log(i_1/i_2)$.

If we consider the simple cases of $f(x)$, where $f(x) = (1-x)^n$, we can determine the reaction order, n , in the following way, using either of the two TG curves employed for determination of the activation energy:

When $f(x) = (1-x)^n$, the conversion integral is given by:

$$F(x) = \int_{x_0}^x dx/(1-x)^n \quad (18)$$

where x_0 is usually chosen as zero. The expression for $F(x)$ when $n \neq 1$ is then:

$$F(x) = (1/(n-1))(1/(1-x)^{n-1}) - 1) \quad (19)$$

If $n = 1$:

$$F(x) = -1n(1-x) = 2.3 \log(1-x) \quad (20)$$

In the subsequent calculations the pre-exponential factor is assumed to be constant, although this does not mean that the results should necessarily be given in terms of A remaining constant in the entire reaction range.

Now, for two points on the same TG curve, we may write:

$$F(x_1) = (A/a)i(E_1, T_1); \quad F(x_2) = (A/a)i(E_2, T_2) \quad (21)$$

$$F(x_1)/F(x_2) = i(E_1, T_1)/i(E_2, T_2) \quad (22)$$

assuming, as noted above, A to be constant in the conversion region x_1 to x_2 .

Eq. (22) can be solved for n using a simple graphical procedure. Thus, one chooses x_2 to be a constant reference conversion, such as 0.1, 0.5, or 0.9, and calculates the ratio $\log(F(x)/F(x_2))$ for a range of values for the conversion x . This ratio is then plotted against n , on the basis of Eq. (19). The apparent order for a given conversion x_1 can be obtained from the graph as the value of n corresponding to $x = x_1$ and $\log(F(x_1)/F(x_2)) = \log(i(E_1, T_1)/i(E_2, T_2))$. The right hand side of Eq. (22) can be calculated on the basis of the tabulated $\log i$ values.

Having determined the analytical form of $f(x)$ (and $F(x)$), the pre-exponential factor can be calculated from the rearranged form of Eq. (11):

$$A = (a/i)F(x) \quad (23)$$

or
$$\log A = \log a - \log i + \log F(x) \quad (24)$$

Calculations and discussion

Values of $\log i$ have been calculated on a computer utilising a 32-point Gaussian numerical integration formula. Their negative logarithms for several temperatures and activation energies are given in Table 1.

Table I
Values of $-(\log i)$

| T, Kelvin | E, kcal/mol | | | | | | | | | |
|-----------|-------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----|-----------|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | |
| 260 | 16.005444 | 24.580245 | 33.107265 | 41.607643 | 50.090994 | 58.562518 | 67.025343 | 75.530771 | | |
| 280 | 14.741837 | 22.715749 | 30.642112 | 38.541931 | 46.424775 | 54.295821 | 62.158187 | 70.013898 | | |
| 300 | 13.642769 | 21.095851 | 28.501613 | 35.880928 | 43.243319 | 50.593940 | 57.935900 | 65.271218 | | 72.601444 |
| 320 | 12.677647 | 19.674947 | 26.625153 | 33.549009 | 40.455990 | 47.351231 | 54.237829 | 61.117797 | | 67.992517 |
| 340 | 11.823045 | 18.418133 | 24.966351 | 31.488313 | 37.993451 | 44.486877 | 50.971678 | 57.449863 | | 63.922808 |
| 360 | 11.060719 | 17.298238 | 23.489107 | 29.653815 | 35.801747 | 41.937996 | 48.065639 | 54.186677 | | 60.302484 |
| 380 | 10.376247 | 16.293792 | 22.164904 | 28.009947 | 33.838264 | 39.654926 | 45.463001 | 51.264482 | | 57.060741 |
| 400 | 9.758077 | 15.387604 | 20.970912 | 26.528243 | 32.068896 | 37.597923 | 43.118380 | 48.632256 | | 54.140920 |
| 420 | 9.196846 | 14.565746 | 19.888638 | 25.185645 | 30.466021 | 35.734800 | 40.995026 | 46.248684 | | 51.499005 |
| 440 | 8.684881 | 13.816811 | 18.902942 | 23.963278 | 29.007030 | 34.039212 | 39.062861 | 44.079953 | | 49.091849 |
| 460 | 8.215838 | 13.131370 | 18.001308 | 22.845540 | 27.673236 | 32.489390 | 37.297028 | 42.098122 | | 46.894028 |
| 480 | 7.784422 | 12.501556 | 17.173299 | 21.819426 | 26.449062 | 31.067185 | 35.676808 | 40.279899 | | 44.877811 |
| 500 | 7.386180 | 11.920757 | 16.410144 | 20.874002 | 25.321416 | 29.757343 | 34.184789 | 38.605715 | | 43.021470 |
| 520 | 7.017338 | 11.383372 | 15.704414 | 20.000014 | 24.279215 | 28.546957 | 32.806235 | 37.059005 | | 41.306613 |
| 540 | 6.674680 | 10.884628 | 15.049779 | 19.189574 | 23.313016 | 27.425026 | 31.528589 | 35.625655 | | 39.717568 |
| 560 | 6.355442 | 10.420426 | 14.440807 | 18.435917 | 22.414719 | 26.382115 | 30.341082 | 34.293564 | | 38.240901 |
| 580 | 6.057241 | 9.987234 | 13.872815 | 17.733208 | 21.577339 | 25.410091 | 29.234430 | 33.052296 | | 36.865026 |
| 600 | 5.778008 | 9.581986 | 13.341739 | 17.076389 | 20.794821 | 24.501899 | 28.200583 | 31.892804 | | 35.579898 |
| 620 | 5.515940 | 9.202009 | 12.844040 | 16.461050 | 20.061886 | 23.651396 | 27.232527 | 30.807208 | | 34.374979 |
| 640 | 5.269455 | 8.844964 | 12.376617 | 15.883332 | 19.373917 | 22.853201 | 26.324123 | 29.788607 | | 33.247979 |
| 660 | 5.037163 | 8.508790 | 11.936744 | 15.339841 | 18.726851 | 22.102585 | 25.469976 | 28.830939 | | 32.186797 |
| 680 | 4.817835 | 8.191672 | 11.522015 | 14.827581 | 18.117103 | 21.395376 | 24.665321 | 27.928850 | | 31.187283 |
| 700 | 4.610381 | 7.891997 | 11.130296 | 14.343897 | 17.541497 | 20.727874 | 23.905939 | 27.077600 | | 30.244173 |
| 720 | 4.413832 | 7.608331 | 10.759689 | 13.886428 | 16.997208 | 20.096790 | 23.188077 | 26.272971 | | 29.352785 |
| 740 | 4.227322 | 7.339396 | 10.408501 | 13.453067 | 16.481715 | 19.499191 | 22.508388 | 25.511202 | | 28.508945 |
| 760 | 4.050075 | 7.084044 | 10.075217 | 13.041927 | 15.992762 | 18.932449 | 21.863874 | 24.788928 | | 27.708917 |
| 780 | 3.881393 | 6.841247 | 9.758475 | 12.651316 | 15.528323 | 18.394208 | 21.251846 | 24.103124 | | 26.949347 |

| | | | | | | | | | |
|------|----------|----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 800 | 3.720649 | 6.610078 | 9.457047 | 12.279707 | 15.086574 | 17.882343 | 20.669882 | 23.451071 | 26.227213 |
| 820 | 3.567275 | 6.389698 | 9.169829 | 11.925725 | 14.665869 | 17.394939 | 20.115795 | 22.830314 | 25.539792 |
| 840 | 3.420756 | 6.179351 | 8.895816 | 11.588122 | 14.264716 | 16.930261 | 19.587608 | 22.238628 | 24.884615 |
| 860 | 3.280626 | 5.978346 | 8.634099 | 11.265767 | 13.881673 | 16.486735 | 19.083524 | 21.673997 | 24.259445 |
| 880 | 3.146459 | 5.786059 | 8.383851 | 10.957632 | 13.515781 | 16.062930 | 18.601912 | 21.134588 | 23.662248 |
| 900 | 3.017869 | 5.601917 | 8.144318 | 10.662779 | 13.165648 | 15.657541 | 18.141283 | 20.618729 | 23.091167 |
| 920 | 2.894499 | 5.425399 | 7.914808 | 10.380350 | 12.830340 | 15.269377 | 17.700278 | 20.124895 | 22.544510 |
| 940 | 2.776027 | 5.256028 | 7.694693 | 10.109563 | 12.508919 | 14.897346 | 17.277653 | 19.651687 | 22.020727 |
| 960 | 2.662153 | 5.093364 | 7.483392 | 9.849697 | 12.200526 | 14.540450 | 16.872269 | 19.197825 | 21.518396 |
| 980 | 2.552604 | 4.937004 | 7.280375 | 9.600091 | 11.904371 | 14.197769 | 16.483078 | 18.762133 | 21.036211 |
| 1000 | 2.447127 | 4.786578 | 7.085150 | 9.360138 | 11.619727 | 13.868458 | 16.109114 | 18.343528 | 20.572972 |
| 1020 | 2.345490 | 4.641743 | 6.897267 | 9.129275 | 11.345924 | 13.551736 | 15.749490 | 17.941012 | 20.127570 |
| 1040 | 2.247477 | 4.502184 | 6.716308 | 8.906987 | 11.082342 | 13.246886 | 15.403384 | 17.553662 | 19.698984 |
| 1060 | 2.152889 | 4.367607 | 6.541889 | 8.692792 | 10.828411 | 12.953239 | 15.070038 | 17.180627 | 19.286267 |
| 1080 | 2.061541 | 4.237741 | 6.373650 | 8.486248 | 10.583598 | 12.670181 | 14.748749 | 16.821117 | 18.888544 |
| 1100 | 1.973262 | 4.112335 | 6.211260 | 8.286941 | 10.347412 | 12.397138 | 14.438864 | 16.474400 | 18.505002 |
| 1120 | 1.887892 | 3.991155 | 6.054412 | 8.094491 | 10.119396 | 12.133579 | 14.139777 | 16.139795 | 18.134887 |
| 1140 | 1.805283 | 3.873984 | 5.902818 | 7.908541 | 9.899126 | 11.879011 | 13.850926 | 15.816671 | 17.777497 |
| 1160 | 1.725297 | 3.760617 | 5.756211 | 7.728759 | 9.686205 | 11.632973 | 13.571785 | 15.504437 | 17.432178 |
| 1180 | 1.647804 | 3.650868 | 5.614342 | 7.554835 | 9.480263 | 11.395035 | 13.301865 | 15.202546 | 17.098332 |
| 1200 | 1.572684 | 3.544558 | 5.476978 | 7.386482 | 9.280956 | 11.164796 | 13.040709 | 14.910483 | 16.775359 |

When the $\log i$ values were plotted against the activation energy straight lines were obtained. A similar, linear, relationship was found to obtain between $\log i$ and the reciprocal of the absolute temperature. In a recent work [12] we have evaluated these relationships and found that the linearity of the $\log i - E$ and $\log i - 1/T$ plots can be characterised by correlation coefficients better than 0.999991 and 0.999969 respectively.

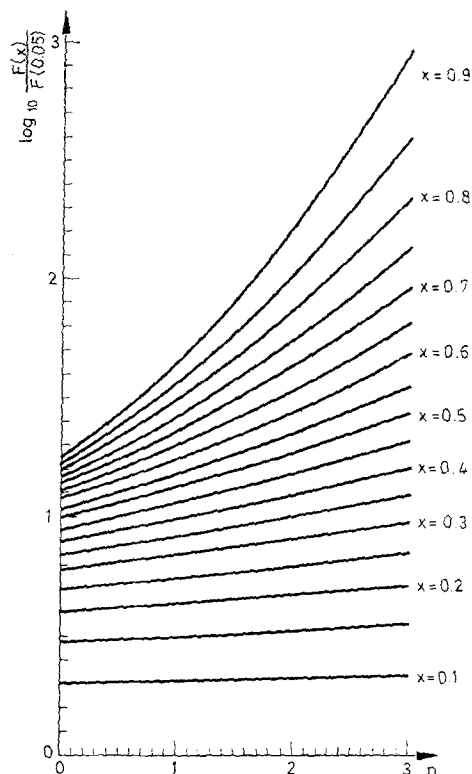


Fig. 1. Curves for determining reaction order. Reference conversion 0.05

These linear relationships are useful in expediting the calculation of activation energy, since it is now sufficient to calculate the $\log i$ values for only two different activation energies; the $\log (i_{11}/i_{21}) - E$ plot can be obtained by simply laying a straight line through these two points. Furthermore, the linearity allows the calculation of $\log i$ values corresponding to any temperature by reciprocal interpolation of data in Table 1. These data are given in such a manner that $\log i$ values can be determined to an accuracy of 10^{-4} per cent by linear interpolation of the reciprocal temperature.

For determination of the reaction order, n , several reference conversions can be used. Figs 1, 2, and 3 are based on reference conversions 0.05, 0.5, and 0.9, respectively, and cover reaction order in the range 0–3. Whenever practicable, it is preferable to use Fig. 3 since with a reference conversion of 0.9 due to the stronger curvature the method is less sensitive to experimental scatter.

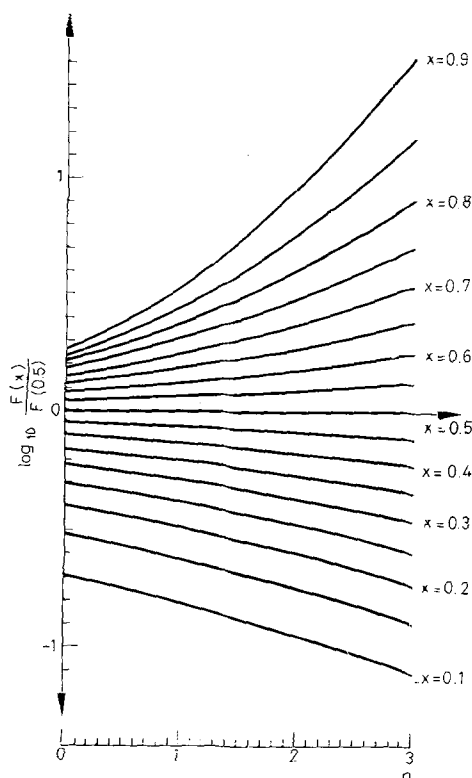


Fig. 2. Curves for determining reaction order. Reference conversion 0.5

To demonstrate the theoretical accuracy of this evaluation method, values of the kinetic constants were calculated from theoretical curves that had been computed in such a way as to restrict the error in the temperature integral to within 10^{-3} per cent. The parameters used to construct these theoretical curves were as follows: $E = 60$ kcal/mol; $n = 1$; $A = 10^{14}$ min $^{-1}$; $a = 2$ and 4 deg/min. The $\log(i_{11}/i_{21}) \sim E$ curves were calculated from $\log i$ values obtained at 50 and 70 kcal/mol activation energies for the given temperatures. For determination of the order, 0.9 was chosen as the reference conversion.

The calculations were carried out on the computer, including the simulation of the graphical procedures to approximate the theoretical accuracy of the method.

The temperature data were fed into the computer with an accuracy of three decimal places.

Results obtained between the conversion limits 0.1 and 0.9 are shown in Table 2. The seventh and eighth columns of Table 2 show a modification of the calculations in which the values of the pre-exponential factor are given in terms of a constant order. This approach is discussed below.

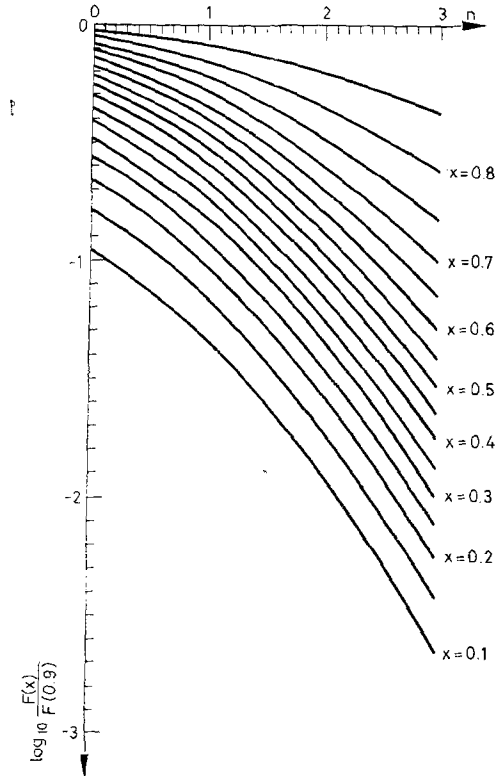


Fig. 3. Curves for determining reaction order. Reference conversion 0.9

Although activation energy, pre-exponential factor, and order of reaction are widely used to characterise processes followed by thermogravimetry and related techniques, there has been some confusion as to the nature of these kinetic parameters. In their methods for the interpretation of thermoanalytical data, Doyle [7, 8] and Zsakó [9] treat all three parameters as constants, while Ozawa [13] assumes the reaction order to be constant and calculates $E - A$ data pairs for points on a master curve.

A serious drawback of the integral evaluation techniques reported in the literature is in the assumption that the kinetic parameters do not change in the

Table 2

Kinetic parameters calculated from theoretical data

| x | T_2, K | T_4, K | $E, \text{kcal/mole}$ | n | $A, \text{min}^{-1} \times 10^{-14}$ | \bar{n} | $A', \text{min}^{-1} \times 10^{-14}$ |
|-----|-----------------|-----------------|-----------------------|--------|--------------------------------------|-----------|---------------------------------------|
| 0.1 | 819.493 | 843.394 | 60.0024 | 1.0006 | 1.002 | 0.9983 | 1.001 |
| 0.2 | 835.649 | 851.134 | 59.9998 | 0.9990 | 0.999 | 0.9983 | 1.000 |
| 0.3 | 846.066 | 861.933 | 59.9994 | 0.9986 | 0.999 | 0.9983 | 1.000 |
| 0.4 | 854.217 | 870.386 | 59.9992 | 0.9984 | 0.999 | 0.9983 | 0.999 |
| 0.5 | 861.264 | 877.696 | 60.0002 | 0.9991 | 1.000 | 0.9983 | 1.000 |
| 0.6 | 867.807 | 884.486 | 59.9975 | 0.9967 | 0.997 | 0.9983 | 0.998 |
| 0.7 | 874.304 | 891.225 | 60.0023 | 1.0009 | 1.002 | 0.9983 | 1.000 |
| 0.8 | 881.314 | 898.507 | 59.9976 | 0.9934 | 0.993 | 0.9983 | 0.997 |
| 0.9 | 890.117 | 907.648 | 60.0014 | — | — | 0.9983 | 0.999 |

Theoretical data: $E = 60$ kcal/mole; $n = 1$; $A = 10^{14} \text{ min}^{-1}$; $a = 2$ and $4^\circ/\text{min}$. A' , column 8, is calculated on the basis of \bar{n} . T_2 and T_4 refer to heating rates of 2 and $4^\circ/\text{min}$, respectively.

integration range. If they do, and the changes are not taken into consideration, results obtained by these techniques will be cumulative ones, and values obtained for a given $x - T$ point will be characteristic only of that point on the TG curve itself, but not of the $x - T$ point in the actual reaction path. Thus results obtained in this way merely describe the TG curve, and the question as to which parameter or parameters should be given as constants loses its relevance.

Techniques assigning single values to the three kinetic parameters are generally less successful from the point of view of curve fitting than those letting at least one of the parameters move freely. The procedures described in this paper for the determination of reaction order and pre-exponential factor can be used with variables modified according to the required form of data interpretation. The seventh and eighth columns of Table 2 show an example of this; here the results are given in terms of a constant reaction order, which has been produced as an average of orders belonging to different conversions.

Compared to integral procedures previously described, ours has several advantages. For example, the activation energy can be determined from point to point; this gives an indication as to whether or not the reaction can be characterised by a single E value. There is no limitation concerning the reaction mechanism, thus complicated reactions can also be investigated by compiling the appropriate graphs. It is possible to establish the order or other parameters of the analytical form of $f(x)$ by a continuous method thus avoiding the time consuming trial-and-error curve-fitting procedures.

The calculations to establish the parameters are carried out by using full integrals instead of approximate equations.

Several workers [13–15] have reported methods for the determination of kinetic parameters which employ two or more TG curves in the evaluation procedure. Ozawa's method [13] is generally regarded as being the most reliable.

In this he utilises Doyle's approximation [4] for the logarithm of the $p(z)$ term in Eq. (8):

$$\log p(z) \approx -2.315 - 0.4567E/RT \quad (25)$$

and obtains the following expression for points of identical conversion on TG curves obtained at different heating rates:

$$d \log a/d \frac{1}{T} \approx 0.4567E/R$$

He then calculates the activation energy from the slope of the $\log a - 1/T$ linear relationship. Doyle [7] obtained the coefficients of Eq. (25) by evaluating the $\log p(z) - E/RT$ relationship over the range $20 < E/RT < 60$. Since this relationship is not completely linear, the use of the coefficients so calculated gives rise to errors in the approximation [12] and, therefore, in any kinetic parameters determined with its aid.

In our method we have assumed that the $\log i - E$ relationship is linear. Errors are insignificant if this assumption is applied to the vicinity of the unknown activation energy.

The superiority of the $\log i - E$ linear approximation over the $\log i - 1/T$ approximation, which is in fact that employed by Ozawa, is discussed elsewhere [12]. However, on statistical grounds, Ozawa's use of more than two TG curves in the evaluation will reduce the error arising from the scatter of experimental results.

An approximate value for the error originating from incorrect temperature measurement or an erroneous a_1/a_2 value can be assessed from a formula obtained by applying Doyle's approximation [7] to the conditions of the present evaluation method:

$$E \approx \log (a_2/a_1)(R/0.4567)(T_1T_2/(T_2 - T_1))$$

This expression shows that deviations from the assumed heating-rate ratio or errors affecting equally the absolute values of T_1 and T_2 give rise only to moderate changes in the calculated activation energy. The method is, however, very sensitive to errors in the difference $T_2 - T_1$. Thus if the value of $T_2 - T_1$ corresponding to a particular a_2/a_1 ratio is 20° and the value obtained experimentally is 19° , the apparent activation energy, E' , is given by:

$$E' = (20/19) E$$

which represents an error of about 5 per cent.

The bigger the difference between the two temperatures the smaller the significance of an error in the temperature reading or of a shift which might be due to the slow diffusion of gaseous products. It is, therefore, advantageous to carry out measurements at heating rates which differ as much as possible, i.e. at the largest practicable a_2/a_1 ratio ($a_2 > a_1$).

To diminish the effects of inadequate removal of gaseous products, low heating rates, small sample size, and the application of vacuum, are recommended.

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RÉSUMÉ — On passe brièvement en revue les méthodes de calcul des énergies d'activation, des facteurs préexponentiels et des ordres de réaction à partir des données thermogravimétriques. On propose une nouvelle méthode d'intégration pour déterminer ces paramètres cinétiques en utilisant les données fournies par des courbes TG groupées par paires et obtenues avec différentes vitesses de chauffage. En employant des valeurs précises de l'intégrale de la température de l'équation d'Arrhenius, fournies par un tableau donnant différentes valeurs de E et de T ou obtenues par un procédé graphique simple, la méthode offre des avantages de rapidité et d'exactitude supérieurs à ceux décrits auparavant. On propose qu'au moins l'un des paramètres cinétiques puisse varier librement afin d'assurer le meilleur ajustement possible entre les courbes expérimentales et calculées.

ZUSAMMENFASSUNG — Eine kurze Übersicht über die Berechnungsmethoden von Aktivierungsenergien, preexponentiellen Faktoren und Reaktionsordnungen aus thermogravimetrischen Daten wird gegeben. Eine neue Integralmethode zur Bestimmung dieser kinetischen Parameter wird vorgeschlagen, die Daten von durch verschiedene Aufheizungsgeschwindigkeiten aufgenommenen Paaren von TG-Kurven verwendet. Durch Anwendung genauer Temperatur-Integralwerte der Arrhenius-Gleichung, welche in einem gewissen E und T Bereich in eine Tabelle eingetragen werden, sowie durch ein einfaches graphisches Verfahren gestattet die Methode Geschwindigkeit und Genauigkeit, die den früher beschriebenen überlegen sind. Die freie Variationsmöglichkeit wenigstens eines der kinetischen Parameter wird vorgeschlagen, um die bestmögliche Anpassung der berechneten und experimentellen Linien zu erhalten.

Резюме — Дан краткий обзор методов расчета энергии активации, предэкспоненциальных факторов и порядка реакции на основе термогравиметрических данных. Предложен новый интегральный метод для определения этих кинетических параметров, используя данные пар кривых ТГ, снятых при различных скоростях нагрева. При применении точных значений температурного интеграла уравнения Аррениуса, стабилизированных по диапазону E и T , и простого графического приема, данный метод, по сравнению с ранее предложенными, имеет преимущества в скорости и точности.